STUDY AID

NLN

TENSORS + INDICAL NOTATION

RECALL THAT A TENSOR (OR VECTOR) IS ITSELF INDEPENDENT OF COOR. SYSTEM BUT CAN BE DESCRIBED IN ANY COORD. SYS. HERE ARE SOME NOTATIONS TO BE COMFORTABLE WITH:

Tensor Notation – $\underset{\sim}{u}$ (vector) or $\underset{\approx}{W}$ (tensor)

THIS IS INDEP. OF COORD SYS.

INDICAL NOTATION – $u_i \underset{\sim}{e_i}$ (vector) or $W_{ij} \underset{\sim}{e_i} \otimes \underset{\sim}{e_j}$

Notes: 1) THIS NOTATION IS COOR. SYS. DEPENDENT. u_i or W_{ij} WILL BE DIFFERENT IN CYLINDRICAL V.S. CARTESIAN, FOR EXAMPLE. OR EVEN ROTATIONS OF CARTESIAN COOR SYS.'S WILL HAVE DIFFERENT COMPONENTS

2) SUMMATION Convention applies here, so $u_i \underset{\sim}{e_i} = u_1 \underset{\sim}{e_1} + u_2 \underset{\sim}{e_2} + u_3 \underset{\sim}{e_3}$

or $W_{ij} \underset{\sim}{e_i} \otimes \underset{\sim}{e_j} = W_{11} \underset{\sim}{e_1} \otimes \underset{\sim}{e_1} + W_{12} \underset{\sim}{e_1} \otimes \underset{\sim}{e_2} + W_{13} \underset{\sim}{e_1} \otimes \underset{\sim}{e_3} \cdots\cdots$ (9 terms)

COMPONENT NOTATION

u_i (vector) or W_{ij} (tensor)

THIS IS THE SAME AS INDICAL NOTATION BUT A SHORTHAND, WHERE YOU DO NOT WRITE OUT THE BASE VECTORS.

NOTES: 1) THIS NOTATION IMPLICITLY ASSUMES A PARTICULAR COOR SYS.

2) W_{ij} = any one of the nine components of $\underset{\sim}{W}$, because i AND j are not repeated indecies. So THIS IS generally REPRESENTING ALL POSSIBLE VALUES $i, j = 1, 2, 3$ INDIVIDUALLY. IN THIS CASE i AND j ARE CALLED "FREE INDECIES".

3) $W_{ii} = W_{11} + W_{22} + W_{33}$
UNLIKE W_{ij}, HERE WE HAVE NO FREE INDECIES, ONLY i REPEATED, IMPLYING SUMMATION FROM 1-3. HERE i IS CALLED A "DUMMY INDEX"

So $[W_{ij}]$ IS ONE OF 9 SCALARS
BUT W_{ii} IS ONE SCALAR $(W_{11}+W_{22}+W_{33})$

MATRIX NOTATION – $\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$ (vector) $\begin{bmatrix} W_{11} & W_{12} & W_{13} \\ W_{21} & W_{22} & W_{23} \\ W_{31} & W_{32} & W_{33} \end{bmatrix}$ (tensor)

Matrix NOTATION IS just A MATRIX REPRESENTATION OF THE COMPONENT NOTATION. HENCE BASE VECTORS ARE IMPLIED BY WHERE YOU ARE IN THE MATRIX. FOR EXAMPLE W_{23} IMPLIES THE SCALAR ASSOCIATED WITH $\underset{\sim}{e_2} \otimes \underset{\sim}{e_3}$. HENCE THIS IS ALSO COOR. SYS. DEPENDENT.

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A POTENTIALLY CONFUSING ANALOGY: CONSIDER AN APPLE  . IT IS OBVIOUSLY THE SAME FRUIT REGARDLESS OF WHAT LANGUAGE YOU SPEAK:

 = APPLE in ENGLISH

 = Pomme in French

 = 苹果 in MANDARIN

(sorry...)

TENSOR $\underset{\sim}{\underset{\sim}{W}}$

COMPONENTS W_{ij}

COORD. SYS $\underset{\sim}{e_i} \otimes \underset{\sim}{e_j}$

NEXT, SOME DEFINITIONS/PROPERTIES:

$$\underset{\sim}{u} \otimes \underset{\sim}{v} \neq \underset{\sim}{v} \otimes \underset{\sim}{u}$$

$$\underset{\sim}{u} \otimes \underset{\sim}{v} \cdot \underset{\sim}{w} = (\underset{\sim}{v} \cdot \underset{\sim}{w})\,\underset{\sim}{u}$$

($\otimes$ disappears b/c dot product reduces this from 2nd to first order tensor, aka vector)

$$\underset{\sim}{w} \cdot \underset{\sim}{u} \otimes \underset{\sim}{v} = (\underset{\sim}{w} \cdot \underset{\sim}{u})\,\underset{\sim}{v}$$

TERM - IN EINSTEN SUMMATION WE SAY

IF AN INDEX APPEARS TWICE IN A "term" THEN SUMMATION IS IMPLIED. A TERM IS ANY SINGLE VARIABLE OR PRODUCT OF VARIABLES.

SO

$$\delta_{ii} = \delta_{11} + \delta_{22} + \delta_{33} \quad (\underline{\text{SUMMATION}})$$
$$= 3$$

BUT $a_i + b_i = a_1 + b_1$ OR $a_2 + b_2$ OR $a_3 + b_3$

i.e. the index is repeated but not in a single term. THIS WOULD BE AN EQUATION IN COMPONENT FORM (above).

FINALLY,

<u>CONTRACTION</u> OF AN EXPRESSION CONTAINING A KRONECKER DELTA IS A SHORT CUT FOR SIMPLIFYNG IT.
TO PERFORM CONTRACTION YOU REWRITE THE EXPRESSION BUT OMIT THE δ AND IN THE REMAINING TERMS, REPLACE ON OF THE δ INDECIES WOT THE OTHER. FOR EXAMPLE,

TO SIMPLIFY $a_i \delta_{i3}$ BY CONTRACTION, WE DROP δ_{i3} AND REPLACE i WITH 3 IN THE REMAINING TERM $\Rightarrow$

$$a_i \delta_{i3} = a_3$$

BUT WE COULD HAVE DONE THIS THE LONGER WAY BY EXPANDING OUT TH SUM AND EVALUATING TO GET THE SAME THING (BUT WITH MORE WORK):

$$
\begin{aligned}
a_i \delta_{i3} &= a_1 \delta_{13} + a_2 \delta_{23} + a_3 \delta_{33} \\
&= a_1 (0) + a_2 (0) + a_3 (1) \\
&= a_3
\end{aligned}
$$